Growth of interfaces with a conservation law and spatial-temporal correlations

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The interfacial growth with a conservation law is analyzed using the dynamic-renormalization-group method. The fixed point is found for the case of long-range spatially and temporally correlated noise. The scaling exponents χ and z are obtained. The result shows that long-range spatial correlations and temporal correlations roughen the interface. The results are compared with those of Medina *et al.* [Phys. Rev. A **39**, 3053 (1989)] and Sun, Guo, and Grant [Phys. Rev. A **40**, 6763 (1989)].

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The dynamics of a growing interface is a challenging problem of both theoretical and practical interest [1–15]. It is closely related to many growing phenomena; examples include layered growth by molecular-beam epitaxy or chemical vapor deposition, development of ordered phase by spinodal decomposition [2], propagation of cluster growth in diffusion-limited aggregation and Eden models [4], etc. Common to these problems is the existence of a surface or interface where active growth occurs. Although complicated patterns appear during the growth, there exist late-time regimes when a dominant large length develops and the growth shows scale invariance. Renormalization-group (RG) techniques, successful in the study of static collective phenomena, have been extended to dynamics and reveal a much more complicated structure of universality classes than the corresponding static case [16–18].

Kardar, Parisi, and Zhang (KPZ) proposed an extremely interesting nonlinear differential equation which gives interfacial-growth exponents consistent with numerical simulations of ballistic aggregation and Eden model in the spatial dimension $d=1$ [5]. Then, Medina, Hwa, Kardar, and Zhang (MHKZ) presented a detailed dynamic-RG analysis of this equation subject to space- and time-correlation noise [11]. After that, Sun, Guo, and Grant (SGG) investigated the dynamics of a growing interface with conservation of total volume under the interface and white noise using both dynamic RG and computer simulation [12]. Recently, RG results for the long-range spatial correlations and conserved growth have been presented by Lam and Family (LF) [14]. In this Brief Report we perform a dynamic-RG analysis on the growing interface with a conservation law and space-time correlations.

We consider the case where the noise has long-range spatial-temporal correlations. The dynamics of a growing interface with a conservation law is described by the model [12]

$$\frac{\partial h}{\partial t}=-\nabla^2\left[\nu\nabla^2 h+\frac{\lambda}{2}(\nabla h)^2\right]+\eta(x,t)\ , \tag{1}$$

which is written in Fourier space as

$$h(k,\omega)=h_0(k,\omega)-\frac{\lambda}{2}G_0(k,\omega)k^2 \times\int_{q,\Omega}\{[\mathbf{q}\cdot(\mathbf{k}-\mathbf{q})]h(q,\Omega) \times h(k-q,\omega-\Omega)\}\ , \tag{2}$$

where

$$h_0(k,\omega)=G_0(k,\omega)\eta(k,\omega)\ , \tag{3}$$

$$G_0(k,\omega)=(-i\omega+\nu k^4)^{-1}\ ,$$

$$\langle\eta(k,\omega)\eta(q,\Omega)\rangle=2D'(k,\omega)k^2\delta^d(k+q)\delta(\omega+\Omega) =2D(k,\omega)\delta^d(k+q)\delta(\omega+\Omega)\ , \tag{4}$$

and $\int_{q,\Omega}=\int d^dq\,d\Omega(2\pi)^{-(d+1)}$. Equation (2) is a convenient starting point for a perturbation calculation of $h(k,\omega)$ in powers of λ as in Ref. [11]. After going through calculations very similar to those of MHKZ, we arrived at the following set of differential recursion relations for the functional renormalization of ν, λ, and $D(k,\omega)$:

$$\frac{d\nu}{dl}=\nu\left[z-4-K_d\frac{\lambda^2 D_1(1)}{\nu^3}\frac{d-4+f_1(1)}{4d}\right]\ , \tag{5}$$

$$\frac{dD(k,\omega)}{dl}=D(k,\omega)\left[z\left[1-\frac{\partial\ln D(k,\omega)}{\partial\ln\omega}\right]-2\chi-d-\frac{\partial\ln D(k,\omega)}{\partial\ln k}\right]+K_d\frac{\lambda^2}{4\nu^3}D_2^2(1)\ , \tag{6}$$

$$\frac{d\lambda}{dl}=\lambda[\chi+z-4]\ , \tag{7}$$

where

$$D_1(1)=\frac{2}{\pi}\int dz\frac{D(1,\nu z)}{(1+z^2)^2}\ , \tag{8}$$

$$f_1(1)=\frac{2}{\pi}\int dz\frac{1}{(1+z^2)^2}\frac{\partial\ln D(k,\omega)}{\partial\ln k}\bigg|_{k=1,\omega=\nu z}\ , \tag{9}$$

$$D_2^2(1)=\frac{2}{\pi}\int dz\frac{D^2(1,\nu z)}{(1+z^2)^2}\ . \qquad (10)$$

We look for a fixed function of the form $D(k,\omega)=D_0+D_\theta k^{-2\rho+2}(\omega/\omega_0)^{-2\theta}$, where the factor $\omega_0=\nu\Lambda^2$ ($\Lambda=1$) is included to make the argument dimensionless. The more complicated case involving nonseparable $D(k,\omega)$ will not be treated here. The relevant integrals needed are evaluated using contour integration. The results are

$$D_1(1)=D_0+D_\theta(1+2\theta)\sec(\pi\theta)\ , \qquad (11)$$

$$D_1(1)f_1(1)=(-2\rho+2)D_\theta(1+2\theta)\sec(\pi\theta)\ , \qquad (12)$$

$$D_2^2(1)=D_0^2+2D_0D_\theta(1+2\theta)\sec(\pi\theta)+D_\theta^2(1+4\theta)\sec(2\pi\theta)\ , \qquad (13)$$

where $0\le\theta\le(\frac{1}{2})$. Expressed in terms of dimensionless parameters $U_0=K_d\lambda^2D_0/\nu^3$ and $U_\theta=K_d\lambda^2D_\theta/\nu^3$, the recursion relations show in the hydrodynamic limit ($k\to0$, $\omega\to0$)

$$\frac{d\nu}{dl}=\nu\left[z-4+U_0\frac{4-d}{2d}+U_\theta\frac{4-d+2\rho-2}{4d}(1+2\theta)\sec(\pi\theta)\right]\ , \qquad (14)$$

$$\frac{d\lambda}{dl}=\lambda[\chi+z-4]\ , \qquad (15)$$

$$\frac{dU_\theta}{dl}=U_\theta[z(1+2\theta)-2\chi-d+2\rho-2]\ , \qquad (16)$$

$$\frac{dU_0}{dl}=U_0(z-2\chi-d)+\frac{U_0^2}{4}+\tfrac{1}{2}U_0U_\theta(1+2\theta)\sec(\pi\theta)+\tfrac{1}{4}U_\theta^2(1+4\theta)\sec(2\pi\theta)\ . \qquad (17)$$

We can solve the fixed point from the recursion relation. The exponents are

$$\chi=\frac{2-d+2\rho+8\theta}{3+2\theta}\ , \qquad (18)$$

$$z=\frac{10+d-2\rho}{3+2\theta}\ . \qquad (19)$$

A stable fixed point in the physical region ($U_0\ge0, U_\theta\ge0$) is found for $0\le\theta\le(\frac{1}{2})$. The situation is changed for $\theta>(\frac{1}{2})$; more divergent terms are generated by RG, giving the fixed function an essential singularity at $\omega=0$. As a consequence the above RG analysis is not used for $\theta>(\frac{1}{2})$.

MHKZ analyzed the KPZ equation subject to space and time correlations in detail. They solved the off-axis fixed point as a function of the dimensionality d and ρ in the presence of spatial correlation in region B [11] and obtain the scaling exponents $z=(4+d-2\rho)/3$ and $\chi=(2-d+2\rho)/3$. They showed that the very-long-range spatial correlations (large ρ) tend to roughen the surface. With temporal correlations, it is hard to give a simple expression because of the absence of Galilean invariance. They proposed a heuristic method, gave some numerical results, and showed that frequency integrals contributing to one-loop propagator and vertex correction will have ultraviolet divergence if the full form of $D^*(\omega)$ [$D^*(\omega)=D_0+D_1\omega^{-2\theta_1}+D_2\omega^{-2\theta_2}$ is the fixed function] is used and a cutoff must be made as $D^*(\omega)$ is really only the behavior in the $\omega\to0$ limit.

SGG considered that the universality classes in nonequilibrium are determined not only by the symmetry of the order parameter and the dimension of space, but also by the presence or absence of conservation laws, mode coupling, and Poisson-bracket relations. They investigated the dynamics of a growing interface with a conservation law and a white noise [12], and found a fixed point $z=(10+d)/3$, $\chi=(2-d)/3$. The hyperscaling relation between χ and z satisfies identity $\chi+z=4$. This is a consequence of the conservation law, which breaks the Galilean invariance as discussed by KPZ.

LF investigated the effects of long-range correlated noise on the dynamics of a conserved growing interface. They gave the fixed-point exponents $\chi=(2-d'+2\rho)/3$, $z=(10+d'-2\rho)/3$ and found for $d'<2$ any amount of spatial correlation is relevant and leads to a universality class different from that of the uncorrelated white noise.

In this report, we have solved for the fixed point in interface growth with a conservation law and long-range spatially and temporally correlated noise. The exponents χ and z are quite similar to those of SGG and LF except for additional spatial and temporal correlation effects. The results shows a fixed point in the region $0\le\theta\le(\frac{1}{2})$ in the hydrodynamic limit. The temporal correlations also tend to roughen the interface like that of long-range spatial correlations. When k and ω become large, it is hard to find a simple expression for χ and z. The result also indicates that in the region $\theta>(\frac{1}{2})$, the dynamic-RG method is no longer applicable for solving such interface growth.

Many investigations still need to be done to gain deep insight into the nature of interface growth with correlations. Numerical simulations are expected to prove the above result.

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